Statistical Characterization of Electromagnetic Wave Propagation in Mine Environments

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***Abstract*—A computational framework for statistically characterizing electromagnetic (EM) wave propagation through mine tunnels and galleries is presented. The framework combines a multi-element probabilistic collocation method with a full-wave fast Fourier transform and fast multipole method accelerated surface integral equation-based EM simulator to statistically characterize fields from wireless transmitters in complex mine environments.**

***Index Terms*—Fast Fourier transform and fast multipole method (FMM-FFT), frequency-domain analysis, integral equations, multi-element (ME), probabilistic collocation (PC), uncertainty quantification, wave propagation in mine environments.**

I. Introduction

UNDERGROUND mines require reliable and effective communication and tracking systems during routine operations and emergency events. The MINER Act of 2006 [1] requires mine operators to implement wireless communication systems capable of surviving disasters and supporting two-way post-event communication and tracking functions. The design of such systems benefits from simulation frameworks capable of statistically characterizing electromagnetic (EM) wave propagation in mine tunnels and galleries occupied by miners and mining equipment (carts, rails, posts, cables) and possibly obstructed by debris from a cave-in. Ideally, these frameworks should provide statistics of key observables (e.g., the probability density function (PDF) of the electric field at a receiver), given uncertainty in the mine geometry, configuration, and excitation (e.g., the positions of transmitters, receivers, and obstacles, and the material properties of mine walls).

Present simulation techniques for analyzing EM wave propagation in mines mostly rely on (multi)modal decomposition [2], [3], ray tracing [4], or cascaded-impedance techniques [5]. These techniques cannot accurately account for presence of miners and equipment, roughness of mine walls (especially when it is comparable to the wavelength), or unstructured debris from a cave-in. At present, none of these methods (except the multimodal based technique in [6]) yields the statistics of pertinent observables. In practice, the statistical characterization of EM wave propagation in realistic mine environments is often carried out experimentally [5], increasingly in active mines [7].

This letter presents a simulation framework that addresses the aforementioned challenges by leveraging a multi-element probabilistic collocation (ME-PC) method to approximate observables (and thereby, their statistics) [8]–[10] over the "random domain" of variables that parameterize the above uncertainties. The proposed framework uses a (deterministic) full-wave fast Fourier transform and fast multipole method accelerated surface integral equation (FMM-FFT-SIE)-based EM simulator to compute observables for various mine configurations represented by selected collocation points in the random domain and subsequently constructs compact multivariate polynomial surrogate models that accurately approximate the observables, yet are computationally cheap to evaluate. Finally, the surrogate models are used in lieu of the computationally expensive EM simulator to extract pertinent statistics via Monte-Carlo (MC).

II. Formulation

Many uncertainties affect the EM wave propagation in mine environments: Examples include the material properties and surface roughness of the (lossy) rock, earth, and mine ore that surround the mine tunnels and galleries; the shape, material composition, and positions of the mining equipment, trolleys, and rails; and the positions and orientations of transmitting and receiving antennas. Assume these uncertainties are parametrized by an $N_{\rm dof}$-dimensional vector $\mathbf{x} = [x^1, x^2, ..., x^{N_{\rm dof}}]$, $\mathbf{x} \in \mathbf{\Omega} = \prod_{i=1}^{N_{\rm dof}} \mathbf{\Omega}^i$, where each random variable x^i resides in domain $\mathbf{\Omega}^i = [a^i, b^i]$.

Let $F(\mathbf{x})$ represent the electric field at a receiver, or any other observable deemed important in the design of a wireless communication network. Its statistical characterization *in principle* could proceed by applying MC, that is by evaluating $F(\mathbf{x})$ for many samples of $\mathbf{x}$, which are selected according to the (assumed/known) probability density functions (PDFs) of the random variables. Unfortunately, MC methods converge very slowly, all but ruling out the use of a CPU-intensive full-wave EM simulator to compute $F(\mathbf{x})$. To circumvent this difficulty, we apply MC methods to an approximate representation (surrogate model) of $F(\mathbf{x})$ constructed using the ME-PC method [8]–[10].

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The ME-PC method leverages generalized polynomial chaos (gPC) expansions to construct a surrogate model from orthogonal polynomials. PC and stochastic collocation (SC) methods are rapidly gaining traction for the purpose of statistically characterizing EM phenomena [11]–[14]. They are nonintrusive, that is they can use existing deterministic simulators to generate surrogate models via multivariate integration rules. Unfortunately, surrogate models generated using classical gPC or SC lack accuracy when the observable exhibits rapid variations across the random domain $\mathbf{\Omega}$, as do electric fields in mine environments. To address this shortcoming, the ME-PC method adaptively and recursively divides the random domain $\mathbf{\Omega}$ into subdomains using the decay rate of the relative error in the gPC expansion as a guide. While constructing a low-order local gPC approximation on each subdomain, the FMM-FFT-SIE EM simulator is used to compute the observable/electric field values at integration (collocation) points dictated by an efficient multivariate integration rule. In what follows, the PC method is reviewed first. The ME-PC method for adaptively constructing local and low-order polynomial approximations to $F(\mathbf{x})$ is explained next.

A. PC Method

The PC method relies on constructing gPC expansion via multivariate orthogonal Legendre polynomials $\mathbf{\Psi}(\mathbf{x})$ as [8], [10]

$$F(\mathbf{x}) \simeq \sum_{m=0}^{N_p} f_m \mathbf{\Psi}_m(\mathbf{x}). \quad (1)$$

Here, $N_p = (N_{\text{dof}} + p)!/(N_{\text{dof}}!p!) - 1$, p is the order of the expansion, and f_m is the mth gPC expansion coefficient expressed as

$$f_m = \int_{\Omega} F(\mathbf{x})\mathbf{\Psi}_m(\mathbf{x})d\mathbf{x}. \quad (2)$$

The N_{dof}-dimensional integral in (2) is evaluated numerically using tensor product (TP) or sparse grid (SG) integration rules [8], [10]. The reason for selecting Legendre polynomials to expand $F(\mathbf{x})$ is that they lead to exponential/optimal convergence of the gPC expansion in (1) for random variables that are uniformly distributed as assumed in this study.

B. ME-PC Method

The gPC expansion in (1) becomes inefficient when the observable varies rapidly in the random variables since it requires polynomials of very high order to yield sufficient accuracy. The ME-PC method circumvents this bottleneck by recursively and adaptively dividing the initial random domain $\mathbf{\Omega}$ into subdomains $\tilde{\mathbf{\Omega}}$, $\tilde{\mathbf{\Omega}} = \prod_{i=1}^{N_{\text{dof}}} \tilde{\mathbf{\Omega}}^i$, with $\tilde{\mathbf{\Omega}}^i = [\tilde{a}^i, \tilde{b}^i]$, using the decay rates of $F(\mathbf{x})$'s local variances as a guide [9], [10], and using the gPC expansion in (1) with low-order polynomials within each subdomain to locally approximate $F(\mathbf{x})$. The local variance of $F(\mathbf{x})$ in $\tilde{\mathbf{\Omega}}$ is approximated by that of the pth-order gPC expansion as

$$\text{var}_p[F(\mathbf{x})] = \sum_{m=1}^{N_p} \tilde{f}_m^2. \quad (3)$$

Here, $\tilde{f}_m$ is the mth coefficient of the gPC expansion constructed in $\tilde{\mathbf{\Omega}}$ and is computed using (2). The decay rate of the local gPC expansion's relative error is defined as $\gamma = (\text{var}_p[F(\mathbf{x})] - \text{var}_{p-1}[F(\mathbf{x})])/\text{var}_p[F(\mathbf{x})]$. The subdomain $\tilde{\mathbf{\Omega}}$ is selected for adaptive refinement in case the subdomain refinement criterion, $\gamma^{\tau_1} J > \varepsilon_1$, is satisfied; here, ε_1 and τ_1 are the desired tolerance and a user-defined constant $(0 < \tau_1 \leq 1)$, respectively; J is the ratio of the volume of $\tilde{\mathbf{\Omega}}$ to that of $\mathbf{\Omega}$. The refinement is performed along only dimensions in which $F(\mathbf{x})$ varies rapidly. The sensitivity of each dimension in $\tilde{\Omega}$ is defined as $\alpha^i = \tilde{f}^i/(\text{var}_p[F(\mathbf{x})] - \text{var}_{p-1}[F(\mathbf{x})])$, where $\tilde{f}^i$ stands for the coefficient of the pth-order gPC expansion that applies to the ith dimension. Refinement is performed along the ith dimension if the dimensional refinement criterion, $\alpha^i \geq \tau_2 \cdot (\max_{i=1,\ldots,N_{\text{dof}}} \alpha^i)$, is satisfied for ith dimension; here τ_2 is a user-defined constant $(0 < \tau_2 \leq 1)$. The adaptive and recursive application of this procedure generates N_d nonoverlapping subdomains $\tilde{\Omega}_j = \prod_{i=1}^{N_{\text{dof}}} \tilde{\Omega}_j^i$, $j = 1, \ldots, N_d$, with $\tilde{\Omega}_j^i = [\tilde{a}_j^i, \tilde{b}_j^i]$, which do not need refinement. Using the gPC coefficients of such subdomains, $\tilde{f}_{m,j}$, the global mean of $F(\mathbf{x})$ over $\mathbf{\Omega}$ can be approximated as

$$E[F(\mathbf{x})] \cong \sum_{j=1}^{N_d} \tilde{f}_{0,j} J_j \quad (4)$$

where $J_j = \prod_{i=1}^{N_{\text{dof}}} [(\tilde{b}_j^i - \tilde{a}_j^i)/(b^i - a^i)]$ is the ratio of the volume of $\tilde{\mathbf{\Omega}}_j$ to that of $\mathbf{\Omega}$ and $f_{0,j}$ denotes the 0th-order coefficient in the pth-order local gPC expansion (in $\tilde{\mathbf{\Omega}}_j$). Similarly, the global variance of $F(\mathbf{x})$ over $\mathbf{\Omega}$ can be computed using

$$\text{var}\,[F(\mathbf{x})] \cong \sum_{j=1}^{N_d} [\sum_{m=1}^{N_p} \tilde{f}_{m,j}^2 + (\tilde{f}_{0,j} - E[F(\mathbf{x})])^2] J_j. \quad (5)$$

The PDF of $F(\mathbf{x})$ can be estimated by applying an MC method to the surrogate model obtained using local gPC expansions generated in N_d nonoverlapping subdomains [8], [10]. It should be noted here that while generating surrogate models of various observables, the refinement parameters γ and α^i, $i = 1, \ldots, N_{\text{dof}}$ are computed for all observables. If one of γ computed for all observables satisfies the above given subdomain refinement criterion, the subdomain $\tilde{\mathbf{\Omega}}$ is selected for refinement. The selected subdomain $\tilde{\mathbf{\Omega}}$ is refined along the ith dimension if one of α^i, $i = 1, \ldots, N_{\text{dof}}$, computed for all observables satisfies the dimensional refinement criterion.

C. FMM-FFT-SIE EM Simulator for Mine Environments

To obtain the coefficients of the gPC expansion constructed in $\tilde{\mathbf{\Omega}}$, the values of $F(\mathbf{x})$ at the integration (collocation) points dictated by the integration rule are computed by FMM-FFT-SIE EM simulator. This simulator characterizes EM wave propagation in electrically very large and realistically loaded mine tunnels by solving Poggio–Miller–Chang–Harrrington–Wu–Tsai and electric field SIEs to account for the scattering from mine walls and perfect electric conducting (PEC) mining equipment (e.g., carts, rails, posts, cables), respectively. To this end, the simulator expands the electric and magnetic currents on surfaces of mine walls and electric currents on surfaces of mining equipment in terms of Rao–Wilton–Glisson (RWG)

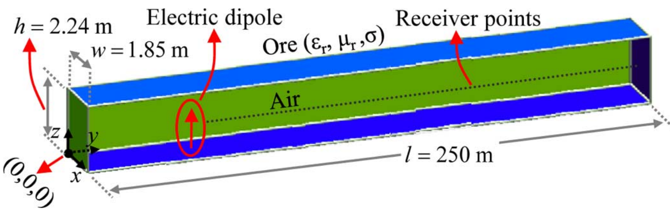

Fig. 1. Geometry of 250-m-long empty rectangular mine tunnel (the lateral wall is removed for illustration).

basis functions [15], then applies a Galerkin testing procedure to obtain a linear system of equations. The solution of this system is obtained using a transpose-free quasi-minimal residual iterative solver. A brute-force application of this solver would require $O(N_{\rm iter}N^2)$ operations; here, $N_{\rm iter}$ stands for the number of iterations, and N denotes the total number of RWG basis functions. This classical simulator, therefore, is prohibitively expensive when applied to the analysis of EM wave propagation in electrically very large and realistically loaded mine tunnels. To alleviate this problem, the computational cost of the iterative solver is reduced to $O(N_{\rm iter}N^{4/3}\log^{2/3}N)$ using an FMM-FFT acceleration scheme that calls for $O(N^{4/3}\log^{2/3}N)$ operations to perform a single matrix-vector multiplication [14], [16], [17].

III. Numerical Results

This section serves to validate the proposed framework when applied to the statistical characterization of EM wave propagation in a mine tunnel populated with carts. In all examples considered here, the mine tunnels are assumed to have smooth walls with relative permittivity $\varepsilon_{\rm r} = 3$, relative permeability $\mu_{\rm r} = 1$, and conductivity $\sigma = 0.001$ S/m. All simulations were carried out on a cluster of dual hexacore X5650 Intel processors located at the Center for Advanced Computing, University of Michigan, Ann Arbor, MI, USA.

A. Validation of the FMM-FFT-SIE EM Simulator

To validate the FMM-FFT-SIE EM simulator, wave propagation in an empty rectangular mine tunnel is characterized (Fig. 1). In this tunnel, an electric dipole with unit moment is positioned at $(0.9, 50, 1)$ m, and the magnitudes of the electric field due to this dipole are computed at receivers located on a line connecting $(0.9, 51, 1)$ m and $(0.9, 250, 1)$ m; different receiver locations are spaced 10 cm apart. The mine walls are discretized using $N = 30\,874\,992$ RWG basis functions. Two different scenarios are considered.

In the first scenario, the electric dipole is z-directed and operated at 455 MHz. Electric field at the receivers is computed by the FMM-FFT-SIE EM simulator. Fig. 2(a) plots the magnitude of electric field's z-component, $|E_z|$, normalized to its maximum, $|E_z|_{\max}$, which is computed at the first receiver (closest to source), and compared to those obtained by the approximate multimodal decomposition method described in [3]. It is apparent from Fig. 2(a) that the results are in good agreement. As the multimodal decomposition method only incorporates z-polarized hybrid modes and ignores the other modes launched from the source, it cannot compute $|E_z|$ values in the near field

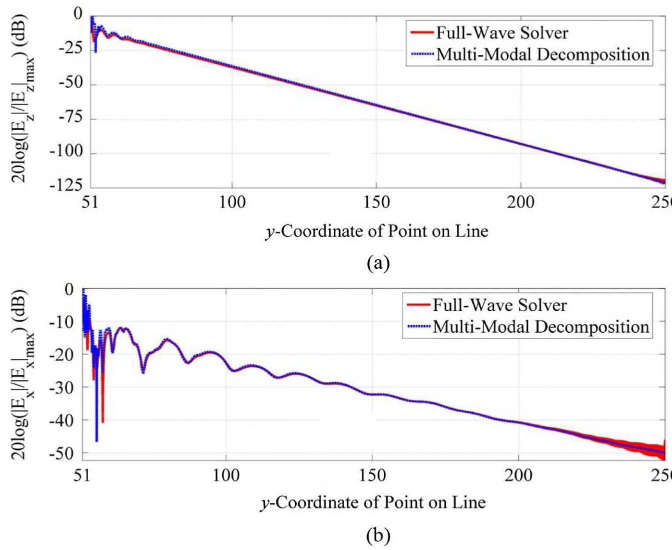

Fig. 2. $20\log(|E_z|/|E_z|_{\max})$ and $20\log(|E_x|/|E_x|_{\max})$ values computed at receiver points by the FFT-FMM-SIE EM simulator and the multimodal decomposition method at (a) 455 and (b) 915 MHz.

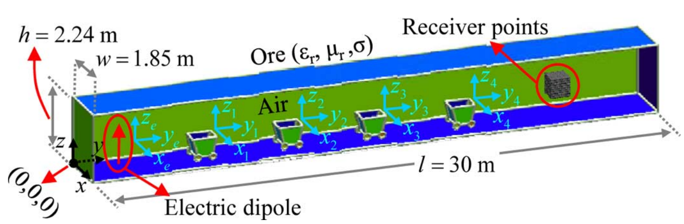

Fig. 3. Geometry of 30-m-long rectangular mine tunnel loaded with carts (the lateral wall is removed for illustration).

of the dipole accurately. Moreover, since the multimodal decomposition formulation neglects the effects of the walls at both ends of the mine tunnel, it cannot capture the standing wave phenomena that occurs near the walls at $y = 0$ and $y = 250$ m.

In the second scenario, the electric dipole is x-directed and operated at 915 MHz. Electric field at the receivers is computed by the FMM-FFT-SIE EM simulator. Fig. 2(b) plots the magnitude of electric field's x-component, $|E_x|$, normalized to its maximum, $|E_x|_{\max}$, which is computed at the first receiver, and compared to those obtained by the multimodal decomposition method [3]. Again, the results are in good agreement.

B. Application of the Framework to the Statistical Characterization of EM Wave Propagation in a Loaded Mine Tunnel

Next, the proposed statistical framework is used to statistically characterize EM wave propagation in a rectangular mine tunnel loaded with four PEC mine carts (Fig. 3). The carts are centered at (x_j, y_j, z_j), $j = 1, \ldots, 4$, and modeled as truncated inverted pyramids with base and top cross sections of 0.5×0.6 m^2 and 0.8×0.6 m^2, respectively; the heights of the truncated inverted pyramids are 0.6 m. The wheels of mine carts are modeled as circular plates with a radius of 0.1 m. The tunnel is excited by a z-directed electric dipole (with unit moment) positioned at $(x_{\rm e}, y_{\rm e}, z_{\rm e})$ and operated at 455 MHz. The observables are the magnitudes of the electric field's z-component $F_n(\mathbf{x}) = |E_{z,n}(\mathbf{x})|$ computed at a grid of $n = 1, \ldots, 1331$

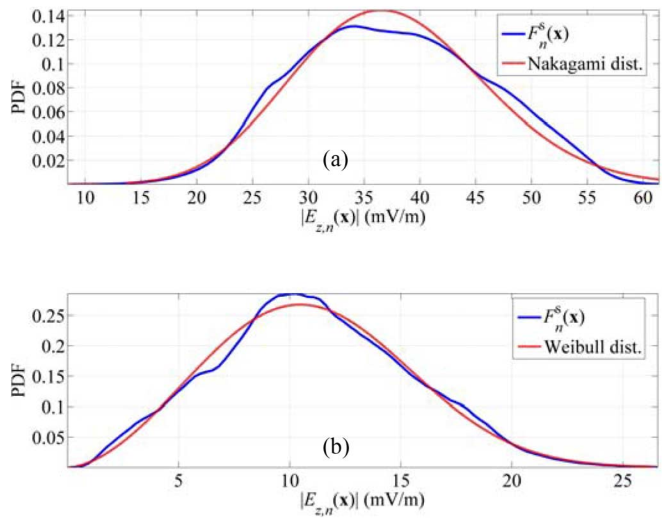

Fig. 4. PDF of $|E_{z,n}(\mathbf{x})|$ (approximated by $F_n^{\mathrm{s}}(\mathbf{x})$) computed at receiver points and its best fitting distribution for scenarios (a) I and (b) II.

receiver points selected in a cube with the side length of 0.6 m and centered at $(0.925, 25.0, 1.12)$ m. The spacing between receiver points along each direction is 0.06 m. Two different scenarios with two different random vectors $\mathbf{x}$ are considered. In both scenarios, the surrogate models $F_n^{\mathrm{s}}(\mathbf{x})$, which approximate $|E_{z,n}(\mathbf{x})|$, $n = 1, \ldots, 1331$, are generated using the ME-PC method with $\varepsilon_1 = 10^{-1}$ and $\tau_1 = \tau_2 = 0.5$. The maximum error in $F_n^{\mathrm{s}}(\mathbf{x})$, $n = 1, \ldots, 1331$, is computed using

$$err = \max_{n=1,\ldots,1331} \sqrt{\frac{\sum_{i=1}^{160} (F_n^{\mathrm{s}}(\mathbf{x}_i) - |E_{z,n}(\mathbf{x}_i)|)^2}{\sum_{i=1}^{160} |E_{z,n}(\mathbf{x}_i)|^2}} \quad (6)$$

where $\mathbf{x}_i$ represents a sample of the random domain. A single PDF accounting for all $F_n^{\mathrm{s}}(\mathbf{x})$, approximating that of all $|E_{z,n}(\mathbf{x})|$, is generated by running a 1-million-point MC on $F_n^{\mathrm{s}}(\mathbf{x})$.

In the first scenario, $\mathbf{x} = [x_{\mathrm{e}}, y_{\mathrm{e}}, z_{\mathrm{e}}]$, where x_{e}, y_{e}, and z_{e} are uniformly distributed in $[0.625, 1.125]$ m, $[0.82, 1.42]$ m, and $[1.0, 1.6]$ m, respectively, and $(x_j, y_j, z_j) = (0.65, 6 + (j - 1) \times 5, 0.55)$ m, $j = 1, \ldots, 4$. $F_n^{\mathrm{s}}(\mathbf{x})$ are constructed via the ME-PC method that uses the TP integration method with five Gauss–Legendre points in each dimension to compute the coefficients of local gPC expansions with $p = 4$. To obtain $F_n^{\mathrm{s}}(\mathbf{x})$, the proposed method required 375 deterministic simulations and $err = 6.4387 \times 10^{-3}$. The PDF of $F_n^{\mathrm{s}}(\mathbf{x})$ reasonably matches the Nakagami distribution [Fig. 4(a)].

In the second scenario, $(x_{\mathrm{e}}, y_{\mathrm{e}}, z_{\mathrm{e}}) = (0.925, 1.0, 1.12)$ m, $\mathbf{x} = [y_1, y_2, y_3, y_4]$, where y_j, $j = 1, \ldots, 4$, are uniformly distributed in $[5.7, 6.3]$ m, $[10.7, 11.3]$ m, $[15.7, 16.3]$ m, and $[20.7, 21.3]$ m, respectively, and $x_j = 0.65$ m and $z_j = 0.55$ m. $F_n^{\mathrm{s}}(\mathbf{x})$ are constructed via the ME-PC method that uses the Gauss–Legendre quadrature based SG integration method (with $l = 2$) (for the definition of l, see [10, Appendix]) to compute the coefficients of local gPC expansions with $p = 2$. To obtain $F_n^{\mathrm{s}}(\mathbf{x})$, the proposed method required 969 deterministic simulations and $err = 5.0691 \times 10^{-2}$. The PDF of $F_n^{\mathrm{s}}(\mathbf{x})$ reasonably matches the Weibull distribution [Fig. 4(b)].

IV. Conclusion

A computational framework leveraging an adaptive ME-PC scheme in conjunction with an FMM-FFT accelerated SIE-based EM simulator for statistically characterizing EM wave propagation in mine tunnels was presented. The technique yields PDFs of receiver field strengths accounting for uncertainty in mine layout and composition and can aid in to assess network reliability during normal operations. Current research focuses on applying the technique to predict the reliability of wireless mine communication systems in the presence of a partial collapse, as well as the hybridization of the above technique with a domain decomposition method to allow for modeling of complete mines involving complex galleries and mile-long tunnels.

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